

Qw 1:

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|----------------------------|----------------------------|
| 1. Second order, linear | 2. Second order, nonlinear |
| 3. Fourth order, linear | 4. First order, nonlinear |
| 5. Second order, nonlinear | 6. Third order, linear |

19. Sol:

$$t^2 \ddot{y} + 4t \dot{y} + 2y = 0 \quad (\cdot := \frac{d}{dt})$$

$$y = t^r \Rightarrow r(r-1)t^r + 4r t^r + 2t^r = 0$$

$$\Rightarrow r^2 + 3r + 2 = 0$$

$$\Rightarrow \boxed{r = -1 \quad \text{or} \quad -2}$$

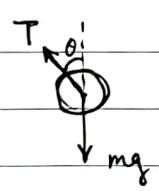
20. Sol:

$$r(r-1) - 4r + 4 = 0$$

$$\Rightarrow \boxed{r = 1 \quad \text{or} \quad 4}$$

29. Sol:

a>



$$b> -\frac{d^2}{dt^2}(L\theta) = g \sin \theta$$

$$\Rightarrow \frac{d^2 \theta}{dt^2} + \frac{g}{L} \sin \theta = 0$$

30. Sol:

$$a> T = \frac{1}{2} m |\vec{v}|^2 = \frac{1}{2} m \left| \frac{d}{dt} L\theta \right|^2 = \frac{1}{2} m L^2 \left| \frac{d\theta}{dt} \right|^2$$

$$b> V = mgh = mgL(1 - \cos \theta)$$

$$c> \frac{d}{dt}(T+V) = 0 \Rightarrow mL^2 \frac{d\theta}{dt} \frac{d^2 \theta}{dt^2} = -mgL \frac{d \cos \theta}{dt}$$

$$\Rightarrow \frac{d^2 \theta}{dt^2} + \frac{g}{L} \sin \theta = 0$$

31. Sol:

$$\begin{aligned} a > \vec{M} &= \vec{r} \times \vec{p} = L m v (-\vec{e}_{(3)}) && (\vec{e}_{(3)} \text{ is the unit vector} \\ &= -L m \frac{d(L\theta)}{dt} (-\vec{e}_{(3)}) && \text{in the direction } \odot) \\ &= mL^2 \frac{d\theta}{dt} \vec{e}_{(3)} \end{aligned}$$

$$\begin{aligned} b > \frac{d\vec{M}}{dt} &= \vec{r} \times m\vec{g} \\ &= -Lmg \sin\theta \vec{e}_{(3)} \end{aligned}$$

$$\Rightarrow mL^2 \frac{d^2\theta}{dt^2} = -Lmg \sin\theta$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{g}{L} \sin\theta = 0$$